Statistical Physics & Condensed Matter Theory I: Exercise

Bosonic coherent states

By definition, bosonic coherent states have an arbitrary number of particles. We can however ask what the *average* occupation numbers are in a given coherent state.

a)

Calculate the average total number of particles $\bar{N}$ in a bosonic coherent state $|\phi\rangle$, where

$$\bar{N} = \frac{\langle\phi|\hat{N}|\phi\rangle}{\langle\phi|\phi\rangle}, \qquad \hat{N} = \sum_i a_i^\dagger a_i.$$

b)

Show that the overlap of the coherent state $|\phi\rangle$ with the occupation number basis state $|n_1, n_2, ...\rangle = \prod_i \frac{(a_i^\dagger)^{n_i}}{\sqrt{n_i!}}|0\rangle$ is

$$|\langle n_1, n_2, ...|\phi\rangle|^2 = \prod_i \frac{(\bar{\phi}_i \phi_i)^{n_i}}{n_i!}$$

(in other words, the occupation numbers of a coherent state are Poisson distributed).

c)

Calculate the variance σ from its definition

$$\sigma^2 = \frac{\langle\phi|\hat{N}^2|\phi\rangle}{\langle\phi|\phi\rangle} - \bar{N}^2.$$

How does the relative width $\sigma/\bar{N}$ behave in the thermodynamic limit $\bar{N} \to \infty$?