

# Statistical Physics & Condensed Matter Theory I: Exercise

## Composition rule for Feynman propagator

The free particle's propagator (in one dimension) from position  $q_i$  at time  $t = 0$  to position  $q_f$  at time  $t$  is given by

$$G_{\text{free}}(q_f, q_i; t) = \left( \frac{m}{2\pi\hbar it} \right)^{1/2} e^{\frac{im}{2\hbar t} (q_f - q_i)^2}.$$

Of course, a particle propagating from  $q_i$  at time  $t_i$  to  $q_f$  at time  $t_f$ , must be somewhere at an intermediate 'between' time  $t_b$ . This means that the propagator must obey the composition identity

$$G_{\text{free}}(q_f, q_i; t_f - t_i) = \int_{-\infty}^{\infty} dq_b G_{\text{free}}(q_f, q_b; t_f - t_b) G_{\text{free}}(q_b, q_i; t_b - t_i).$$

Show explicitly (by performing the necessary Gaussian integration, which you can assume to be convergent) that this equation is fulfilled for any  $t_b$ .