Statistical Physics & Condensed Matter Theory I: Exercise

1 Heisenberg chain with next-nearest-neighbour coupling

Consider a one-dimensional lattice of N sites, with spin operators $\mathbf{S}_m$ defined at each site $m = 1, ..., N$, with periodic boundary conditions $\mathbf{S}_{m+N} \equiv \mathbf{S}_m$. We are interested in the Hamiltonian

$$H = -J_1 \sum_{m=1}^{N} \mathbf{S}_m \cdot \mathbf{S}_{m+1} + J_2 \sum_{m=1}^{N} \mathbf{S}_m \cdot \mathbf{S}_{m+2},$$

in which $J_1, J_2 > 0$, *i.e.* the chain has ferromagnetic nearest-neighbour couplings but *anti*ferromagnetic next-nearest-neighbour couplings.

a)

What are the classical and quantum ground states in the limit $J_1 \gg J_2$ (*i.e.* $J_2/J_1 \to 0$)? Describe in a few words what you think happens in the limit $J_2 \gg J_1$.

b)

We now specialize to the case $J_1 \gg J_2$ (leaving J_2 finite). Using the Holstein-Primakoff transformation, write the effective bosonic theory at large S to leading nontrivial order in S.

c)

Obtain the spectrum explicitly to leading nontrivial order in S (in other words, diagonalize the Hamiltonian by a Fourier transform). Up to what value of J_2/J_1 can you have at least some faith in your calculations ? Explain your reasoning.