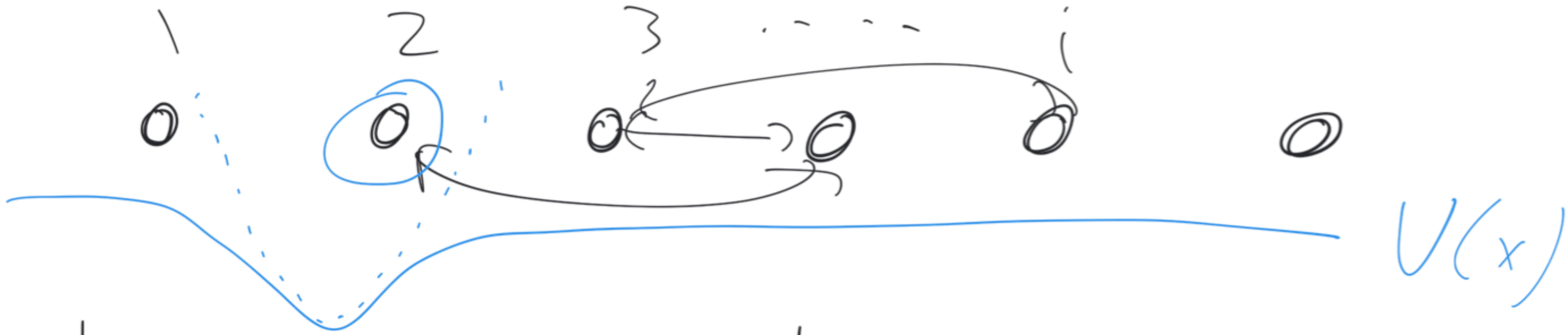
1 Tuesday 5 September



Classical 1D chain

x_i $\quad$ p_i

$$H = \sum_i \frac{p_i^2}{2m} + \frac{k_s}{2}\left(x_{i+1} - x_i - a\right)^2$$

$$L = \sum_i \frac{m}{2}\dot{x}_i^2 - \frac{k_s}{2}\left(x_{i+1} - x_i - a\right)^2$$

1) $X_i = \bar{X}_i + \phi_i(t)$ $\qquad \bar{X}_i = ia$

2) PBC

$$L = \sum_{i=1}^{N} \frac{m}{2}\dot{\phi}_i^2 - \frac{k_s}{2}(\phi_{i+1} - \phi_i)^2$$

3) continuum limit: $\phi_i \to \sqrt{a}\,\phi(x)$
($\lim a \to 0$)

(*) $\phi_{i+1} - \phi_i = a^{\frac{1}{2}}\left(\phi(x+a) - \phi(x)\right) = a^{3/2}\,\partial_x \phi(x)\Big|_{x=ia}$

(*) $\sum \to \frac{1}{a}\int_0^L dx$

$$L[\phi] = \int_0^L dx \ \mathcal{L}(\phi, \dot{\phi}, \partial_x \phi)$$

$$\mathcal{L} = \frac{m}{2} \dot{\phi}^2 - \frac{k_s}{2} a^2 (\partial_x \phi)^2$$

functional

$$S[\phi] = \int dt \ L[\phi]$$

$$\phi(x,t) \longrightarrow \phi(x,t) + \varepsilon\, \eta(x,t)$$

$$\Delta S = S[\phi + \varepsilon \eta] - S[\phi]$$

$$= \int dt \int dx \left(\frac{M}{2}(\dot{\phi} + \varepsilon \dot{\eta})^2 - \frac{M}{2}(\dot{\phi})^2 - \frac{k_s}{2} a^2 \Big[(\partial_x \phi + \varepsilon \partial_x \eta)^2 - (\partial_x \phi)^2 \Big] \right)$$

$$\lim_{\varepsilon \to 0} \frac{\Delta S}{\varepsilon}$$

$$\Delta S = \varepsilon \int dt \int dx \left(m \dot{\phi} \dot{\eta} - k_s a^2 \partial_x \phi \, \partial_x \eta \right) + O(\varepsilon^2)$$

$$= -\varepsilon \int dt \int dx \left(\underline{m \ddot{\phi} - k_s a^2 \partial_x^2 \phi} \right) \eta + \cancelto{0}{\text{Boundary}} + O(\varepsilon^2)$$

$$\Delta S = 0 \quad \forall \, \eta(x,t)$$

$$\boxed{m\ddot{\phi} - k_s a^2 \, \partial_x^2 \phi = 0}$$ e.o.m.

$$\left(m\partial_t^2 - k_s a^2 \partial_x^2\right)\phi = 0 \qquad \forall (x,t)$$

$$\phi(x,t) = \phi_+(x-vt) + \phi_-(x+vt)$$

$$v = a\sqrt{k_s/m}$$