② Energy

$$\Pi_i = m\dot{\phi}_i \qquad \Pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

$$H = \sum_i \frac{\Pi_i^2}{2m} + \frac{k_s}{2}\left(\phi_{i+1} - \phi_i\right)^2$$

$$\rightarrow H[\Pi, \phi] = \int dx \left(\frac{\Pi^2}{2m} + \frac{k_s a^2}{2} (\partial_x \phi)^2 \right)$$

e.o.m: $H = \int dx \; k_s a^2 (\partial_x \phi)^2$

intermezzo

$$u = \frac{1}{L}\langle H \rangle = -\frac{1}{L}\,\partial_\beta \ln Z(\beta)$$

$$Z = \int \prod_i \left(d\tau_i\, d\phi_i \right) \quad e^{\beta H}$$

$$C = \partial_T u$$

$$u = \rho T \Rightarrow \quad c = \rho = cst$$

1D Quantum chain

$$[\hat{\phi}(x), \hat{\pi}(x')] = i\hbar\,\delta(x-x')$$

$$\hat{\mathcal{H}}(\hat{\phi}, \hat{\pi}) = \frac{1}{2m}\hat{\pi}^2(x) + \frac{k_s a^2}{2}(\partial_x \hat{\phi}(x))^2$$

$$\hat{H} = \int dx\, \hat{\mathcal{H}}$$

$$\hat{H} = \int_0^L dx \;\; \frac{1}{2m}\hat{\Pi}(x)\hat{\Pi}(x) + \frac{k_s a^2}{2}(\partial_x\hat{\phi})(\partial_x\hat{\phi})$$

$$= \frac{1}{L}\sum_{k k'}\int_0^L dx \left(\frac{1}{2m}\hat{\Pi}_k \hat{\Pi}_{k'} e^{-i(k+k')x} \right.$$

$$\left. + \frac{k_s a^2}{2}(ik)(ik')\hat{\phi}_k \hat{\phi}_{k'} e^{i(k+k')x} \right)$$

$$\frac{1}{2}\int_0^L dx\, e^{i(k-k')x} = \delta_{k,k'}$$

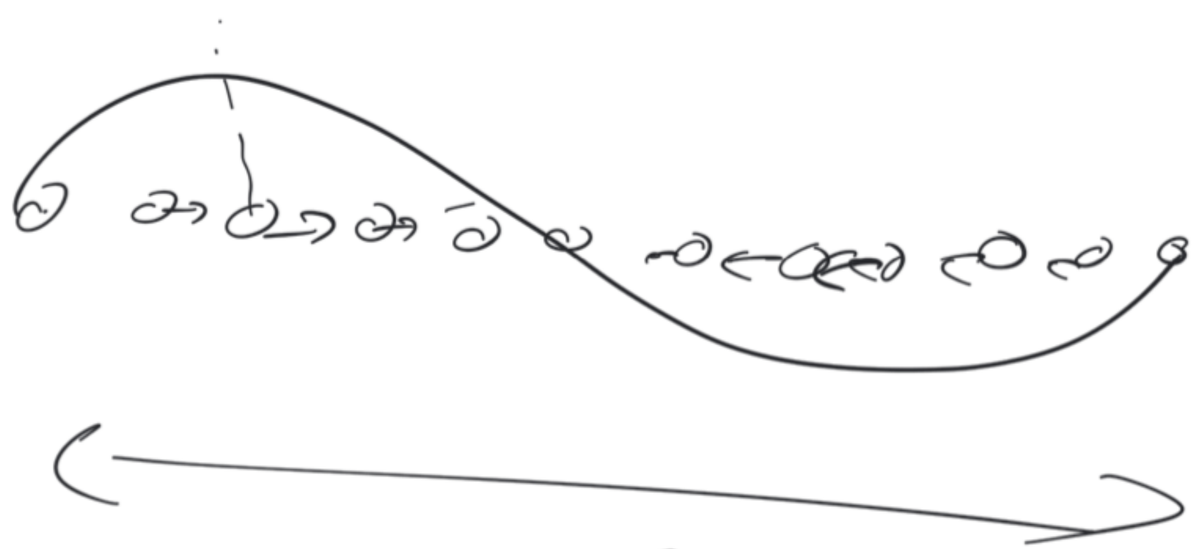
$$\Rightarrow \hat{H} = \sum_k \left(\frac{1}{2m}\hat{\Pi}_k \hat{\Pi}_{-k} + \frac{k_s a^2}{2} k^2 \hat{\phi}_k \hat{\phi}_{-k} \right)$$

$$= \sum_k \frac{1}{2m}|\pi_k|^2 + \frac{k_s a^2}{2}k^2 |\phi_k|^2$$

$$\hat{\phi}_k = \frac{1}{L}\int_0^L e^{-ikx}\,\hat{\phi}(x)$$

$$k = \frac{2\pi}{\lambda}$$

Re



λ

$$\hat{a}_k = \sqrt{\frac{m\omega_k}{2}}\left(\hat{\phi}_k + \frac{i}{m\omega_k}\hat{\Pi}_{-k}\right)$$

$$\omega_k = v|k|$$

$$v = a\sqrt{k_s/m}$$

$$\hat{a}_k^\dagger = (\hat{a}_k)^\dagger \Rightarrow [a_k, a_{k'}^\dagger] = \delta_{kk'}$$

$$[a_k, a_{k'}] = 0$$

Phonons!

$$\hat{H} = \sum_u \omega_u \left(\hat{a}_u^\dagger \hat{a}_u + \frac{1}{2} \right)$$

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intermezzo

$$C = \text{cst}\, T^d$$